The semiclassical tunneling probability in quantum cosmologies with varying speed of light

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Abstract

In quantum cosmology the closed universe can spontaneously nucleate out of the state with no classical space and time. The semiclassical tunneling nucleation probability can be estimated as $P \sim \exp(-\alpha^2/\Lambda)$ where α=const and Λ is the cosmological constant.

In classical cosmology with varying speed of light $c(t)$ (VSL) it is possible to solve the horizon problem, the flatness problem and the Λ-problem if $c = sa^n$ with s=const and $n < -2$. We show that in VSL quantum cosmology with $n < -2$ the semiclassical tunneling nucleation probability is $P \sim \exp(-\beta^2\Lambda^k)$ with β=const and $k > 0$. Thus, the semiclassical tunneling nucleation probability in VSL quantum cosmology is very different from this one in quantum cosmology with c=const. In particular, this one is strongly suppressed for large values of Λ.

1 Introduction.

Among the most interesting alternatives to inflation in physics nowadays are truly the cosmological models with varying speed of light (VSL)[1], [2] [1]. In simplest case the speed of light $c = c(t)$ varies as some power of the expansion scale factor: $c(t) = sa^n(t)$, where constant $s > 0$. Summarizing some of the promising positive features of these models:
1. It can solve the horizon problem if $n < 0$.
2. It can solve the flatness problem in a radiation-dominated early universe if $n < -1$.
3. In case of $n < -2$ the VSL models can solve the Λ-problem in a radiation-dominated early universe while inflation models can't handle it without the aid of the anthropic principle.

Of course, these VSL models result in some shortcomings and unusual (unphysical?) features as well [3]:
1. It is not clear how to solve the isotropy problem.
2. The quantum wavelengths of massive particle states and the radii of primordial black holes can grow sufficiently fast to exceed the scale of the particle horizon.
3. The entropy problem: Entropy can decrease with increasing time.

[1] In fact, there are many articles about this matter. But we'd like to restrict ourselves to consider only those ones which has been used in this work.

Keeping in mind all the above-mentioned problems we'd like, nevertheless, to consider VSL quantum cosmology. There are known the three ways to describe quantum cosmology: the Hartle-Hawking wave function [4], the Linde wave function [5], and the tunneling wave function [6]. In the last case the universe can tunnel through the potential barrier to the regime of unbounded expansion with semiclassical probability $P \sim \exp(-\alpha^2/\Lambda)$. For the universe filled with radiation this result can be obtained in framework of a standard WKB wave function for a particle in a potential $U(a) = c^2a^2(1 - \Lambda a^2/3)$, with the small energy E, see Fig.1. We have two Lorentzian regions ($0 < a < a_i'$, $a > a_i$) and one Euclidean region ($a_i' < a < a_i$). The second turning point $a = a_i$ corresponds to the beginning of our universe. If $\Lambda = 0$ then $U(a)$ has the form of **parabola** and we get only one Lorentzian region (see Fig.1). In this case, the universe can start at $a = 0$, expand to a maximum radius and then recollapse. If $E \to 0$ then the single Lorentzian region is contract to the point. This, of course, comes to an agreement with the tunnelling nucleation probability: $P \to 0$ as $\Lambda \to 0$.

In this article, however, we'll show that in quantum cosmological VSL models the situation can be opposite, viz: the probability to find the finite universe short after it's tunneling through the potential barrier is $P \sim \exp(-\beta(n)\Lambda^{\alpha(n)})$ with $\alpha(n) > 0$ and $\beta(n) > 0$ when $n < -2$ or for $-1 < n < -2/3$. After the tunneling one get the finite universe with "initial" value of scale factor $a_i \sim \Lambda^{-1/2}$, so the probability to find the universe with large value of Λ and small value of a_i is strongly suppressed. The reason of this is that, for the case $\Lambda \to 0$, the potential $U(a)$ is transformed into the **hyperbola** which is located under the abscissa axis and thus such a universe can start at $a \sim 0$ the regime of unbounded expansion (see Fig.2). As a result, we get the single Lorentzian region which is not contract to the point at $E \to 0$.

2 Albrecht-Magueijo-Barrow VSL model

Lets start with the Friedmann and Raychaudhuri system of equations with $k = +1$ (we assume that G=const):

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3},$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \left(\frac{c}{a}\right)^2 + \frac{\Lambda c^2}{3}, \tag{1}$$

$$c = c_0\left(\frac{a}{a_0}\right)^n = sa^n, \qquad p = wc^2\rho,$$

where $a = a(t)$ is the expansion scale factor of the Friedmann metric, p is the fluid pressure, ρ is the fluid density, k is the curvature parameter, Λ is the cosmological constant, c_0 is the modern value of speed of light (3×10^{10} sm/sec) and a_0 is the modern value of the scale factor ($a_0 \sim 10^{28}$ sm).

Using (1) one get

$$\dot{\rho} = -\frac{3\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) + \frac{\dot{c}c(3 - a^2\Lambda)}{4\pi G a^2}. \tag{2}$$

Choosing $w = 1/3$ one can solve (2) to receive

$$\rho = \frac{M}{a^4} + \frac{3s^2na^{2(n-1)}}{8\pi G(n+1)} - \frac{s^2n\Lambda a^{2n}}{8\pi G(n+2)}, \tag{3}$$

where $M > 0$ is a constant characterizing the amount of radiation. It is clear from the (21) that the flatness problem can be solved in a radiation-dominated early universe by an interval of VSL evolution if $n < -1$, whereas the problem of Λ-term can be solved only if $n < -2$. The evolution equation for the scale factor a (the second equation in system (1)) can be written as

$$p^2 + U(a) = E, \qquad E \geq 0, \tag{4}$$

where $p = -a\dot{a}$ is the momentum conjugate to a, $E = 8\pi GM/3$ and

$$U(a) = \frac{s^2 a^{2n+2}}{n+1} - \frac{2s^2 \Lambda a^{2n+4}}{3(n+2)}. \tag{5}$$

The potential (5) has one maximum at $a = a_e = \sqrt{3/(2\Lambda)}$ such that

$$U_e \equiv U(a_e) = \frac{s^2 3^{n+1}}{2^{n+1}\Lambda^{n+1}(n+1)(n+2)}, \tag{6}$$

so $U_e > 0$ if (i) $n < -2$ or (ii) $n > -1$. The first case allows us to solve the flatness and "Lambda" problems. The surplus dividend of the model is the presence of finite time region during which universe has accelerated expansion (see Appendix).

3 The semiclassical tunneling probability in VSL models with $n < -2$: the case $E \ll U_e$

One can choose $n = -2 - m$ with $m > 0$. Such a substitution gives us the potential (5) in the form

$$U_m(a) = \frac{s^2}{a^{2(m+1)}} \left(\frac{2\Lambda a^2}{3m} - \frac{1}{m+1} \right). \tag{7}$$

The equation (4) is quite similar to equation for the particle of energy E that is moving in potential (7), hence the universe in quantum cosmology can start at $a \sim 0$, expand to a maximum radius a_i' and then tunnel through the potential barrier to the regime of unbounded expansion with "initial" value $a = a_i$, see Fig.2. The semiclassical tunneling probability can be estimated as

$$P \sim \exp\left(-2 \int_{a_i'}^{a_i} |\, \tilde{p}(a) \,| da \right), \tag{8}$$

with

$$|\, \tilde{p}(a) \,| = \frac{c^2(t)}{8\pi G\hbar} \,|\, p(a) \,|, \qquad |\, p(a) \,| = \sqrt{U_m(a) - E}, \qquad E \leq U_e.$$

It is convenient to write $E = U_e \sin^2\theta$, with $0 < \theta < \pi/2$.

For the case $E \ll U_e$ one can choose

$$a_i' \sim a_1 = \sqrt{\frac{3m}{2(m+1)\Lambda}}, \qquad a_i \sim \sqrt{\frac{3}{2\Lambda}} \left(\frac{\sqrt{m+1}}{\sin\theta} \right)^{1/m}, \tag{9}$$

and evaluate the integral (8) as

$$P \sim \exp\left(\frac{-s^3 \Lambda^{2+3m/2} I_m(\theta)}{4\pi G\hbar} \right), \tag{10}$$

where

$$I_m(\theta) = \int_{z_i'(\theta)}^{z_i(\theta)} dz z^{-5-3m} \sqrt{\frac{2z^2}{3m} - \frac{1}{m+1}}, \tag{11}$$

with

$$z_i'(\theta) = \sqrt{\frac{3m}{2(m+1)}}, \qquad z_i(\theta) = \sqrt{1.5} \left(\frac{(m+1)^{1/2}}{\sin\theta} \right)^{1/m}.$$

The integral (11) can be calculated for the $m \in Z$. For example

$$I_1(\theta) = \frac{\sqrt{3}}{17010} \left(3 + \cos^2\theta\right) \left(191 - 78\cos^2\theta + 15\cos^4\theta\right) \sqrt{6 + 2\cos^2\theta} \sim 0.148 + O(\theta^6),$$

$$I_2 \sim 0.025, \qquad I_3 \sim 0.007, \qquad I_4 \sim 0.002,$$

and so on. One can further show that $I_m(\theta) > 0$ at $0 < \theta \ll 1$. Thus, it is easy to see from (16) that the semiclassical tunneling probability $P \to 0$ for large values of $\Lambda > 0$ and $P \to 1$ at $\Lambda \to 0$.

Note, that the case c=const can be obtained by substitution $m = -2$ into the (16). Not surprisingly, this case will get us the well known result $P \sim \exp(-1/\Lambda)$ (see [7]).

In quantum cosmology the closed universe can spontaneously nucleate out of a state with no classical space and time. It mean that we must choose

$$(a_i')^2 \leq a_{Pl}^2(t) = \frac{\hbar G}{c^3(t)}. \tag{12}$$

Substituting (9) into (12) one get the inequality,

$$\Lambda \leq \Lambda_m = \frac{3m}{2(m+1)} \left(\frac{\hbar G}{c_0^3 a_0^{3(m+2)}} \right)^{2/(4+3m)}, \tag{13}$$

where c_0 is the modern value of speed of light and a_0 the modern value of scale factor (or the size of visible universe: $a_0 \sim 10^{28}$ sm). For example

$$\Lambda_1 = 0.14\times10^{-90}\ \text{sm}^{-2},\ \Lambda_2 = 0.48\times10^{-80}\ \text{sm}^{-2},\ \Lambda_3 = 0.22\times10^{-74}\ \text{sm}^{-2},\ \Lambda_{10} = 10^{-63}\ \text{sm}^{-2}.$$

For the case $m \ll 1$ we get

$$\frac{4\Lambda^2 c_0^3 a_0^6}{9\hbar G} \leq m^2 \ll 1,$$

so $\Lambda \ll 0.16 \times 10^{-116}$ sm^{-2}.

It is convenient to introduce the cosmological parameter $\Omega_0 =\mid \rho_{\Lambda}(t_0) \mid / \rho_c(t_0)$, where $\rho_{\Lambda}(t_0)$ is the contribution of Λ to modern value of density and $\rho_c(t_0) = 10^{-29}$ gramme/sm^3 is the critical density. One get

$$\Omega_0 = \left| \frac{\rho_{\Lambda}(t_0)}{\rho_c(t_0)} \right| = \frac{(m+2)c_0^2\Lambda}{8\pi G m \rho_c(t_0)} \leq \Omega_m = \frac{(m+2)c_0^2\Lambda_m}{8\pi G m \rho_c(t_0)}.$$

So

$$\Omega_1 = 0.2 \times 10^{-34}, \qquad \Omega_2 = 0.5 \times 10^{-24}, \qquad \Omega_{10} = 0.6 \times 10^{-7}, \qquad \Omega_{20} = 1.3 \times 10^{-4},$$

and so on. Hence, in framework of this model, the whole contribution of Λ in modern era is a quite negligible quantity (And that's just how it should be.).

The obtained values of Λ are seemingly unnatural. Of course, we can assume that $a'_i > a_{Pl}$, in spite of the (12). In this case, we receive the following picture: there is the pre-Big-Bang universe which can start its evolution at $a \sim a_{Pl}$, expand to a maximum radius a'_i and then tunnel through the potential barrier to the regime of unbounded expansion i.e. turn into the universe we see nowadays. The probability of this process will be large for small value of Λ. But the Λ-term in our universe will be the same as in pre-Big-Bang universe. Thus, the above scenario doesn't explain the reason why the Λ was so small in pre-Big-Bang universe and how the pre-Big-Bang universe was born. Therefore, if we want to describe the quantum nucleation of universe we must use the (12). As we have seen, the requirement for this is the verity of (13), i.e. the smallness of Λ and it is highly uncommon that for such values of Λ, the probability of quantum nucleation, in fact, will be so large.

4 The semiclassical tunneling probability with $n < 2$ and $n > -1$

In the case of general position the semiclassical tunneling probability with $n = -2 - m$ has the form

$$P_m \sim \exp\left(-\frac{s^3\Lambda^{(3m+4)/2}}{4\pi G\hbar 3^{(m+1)/2}\sqrt{m(m+1)}}\int_{z'_i}^{z_i}\frac{dz}{z^{3m+5}}\sqrt{F_m(z,\theta)}\right), \tag{14}$$

where

$$F_m(z,\theta) = -2^{m+1}\sin^2\theta\, z^{2(m+1)} + 2\times 3^m(m+1)z^2 - m3^{m+1}, \tag{15}$$

z is dimensionless quantity and z'_i, z_i are the turning points, i.e. two real positive solutions of the equation $F_m(z,\theta) = 0$ for the given θ (it is easy to see that the equation $F_m(z,\theta) = 0$ does have two such solutions at $0 < \theta < \pi/2$).

If m is the natural number then the expression (14) has a more simple form. For example

$$P_1 \sim \exp\left(-\frac{s^3\Lambda^{7/2}\sin\theta}{6\pi G\hbar\sqrt{2}}\int_{z'_i}^{z_i}\frac{dz}{z^8}\sqrt{(z^2 - z_i'^2)(z_i^2 - z^2)}\right), \tag{16}$$

with

$$z'_i = \frac{\sqrt{3}}{2\cos(\theta/2)}, \qquad z'_i = \frac{\sqrt{3}}{2\sin(\theta/2)}.$$

This expression can be calculated exactly:

$$P_1 \sim \exp\left(-\frac{s^3\Lambda^{7/2}\sin\theta J(\theta)}{6\sqrt{2}\pi G\hbar}\right), \tag{17}$$

with

$$J(\theta) = \frac{1}{105}\left(\frac{2\sin(\theta/2)}{\sqrt{3}}\right)^5\left[\frac{8\lambda^4 - 13\lambda^2 + 8}{\cos^2(\theta/2)}\Pi\left(\mu^2; \frac{\pi}{2}\backslash\arcsin\mu\right) - 2\left(2\lambda^4 - \lambda^2 + 2\right)\mathrm{K}(\mu^2)\right],$$

where $\mu^2 = \cos\theta/\cos^2(\theta/2)$, $\lambda = \cot(\theta/2)$, Π and K are complete elliptic integral of the first and the third kinds correspondingly [8].

Similarly,

$$P_2 \sim \exp\left(-\frac{s^3\Lambda^5 \sin\theta}{18\pi G\hbar}\int_{z_i'}^{z_i}\frac{dz}{z^{11}}\sqrt{(z^2+z_1^2)(z^2-z_i'^2)(z_i^2-z^2)}\right), \tag{18}$$

where

$$z_1 = \sqrt{\frac{3}{\sin\theta}\cos\left(\frac{\theta}{3}-\frac{\pi}{6}\right)}, \qquad z_i' = \sqrt{\frac{3}{\sin\theta}\sin\frac{\theta}{3}}, \qquad z_i = \sqrt{\frac{3}{\sin\theta}\cos\left(\frac{\theta}{3}+\frac{\pi}{6}\right)},$$

and so on.

Therefore the probability to obtain (via quantum tunneling through the potential barrier) the universe in the regime of unbounded expansion is strongly suppressed for large values of Λ and small values of the initial scale factor $a_i = \sqrt{3}/(2\sin(\theta/2)\sqrt{\Lambda})$. In other words, overwhelming majority of universes which are nascent via quantum tunneling through the potential barrier (5) have large initial scale factor and small value of Λ. Furthermore, it's clear that this case permits to obtain the analog of the (13) too.

Now, let us consider the case (ii), when $n > -1$. The "quantum potential" has the form

$$U(a) = s^2 a^{2m}\left(\frac{1}{m} - \frac{2\Lambda a^2}{3(m+1)}\right), \tag{19}$$

where $m = n+1 > 0$, see Fig. 3. The points of intersection with the abscissa axis a are $a_0 = 0$ and $a_1 = \sqrt{3(m+1)/2\Lambda m}$. Choosing $E = 0$ in equation (4) and substituting (19) into the (8) we get

$$P \sim \exp\left(-\frac{s^3\Lambda^{(1-3m)/2}}{4\pi G\hbar}\int_0^{z_1} z^{2m-2}\sqrt{\frac{1}{m}-\frac{2z^2}{3(m+1)}}\,dz\right), \tag{20}$$

with $z_1 = \sqrt{3(m+1)/2m}$ [2]. Thus, we have the same effect as if $0 < m < 1/3$.

The density ρ is

$$\rho = \frac{M}{a^4} - \frac{3(1-m)s^2}{8\pi G m a^{4-2m}} + \frac{s^2(1-m)\Lambda}{8\pi G(1+m)a^{2-2m}}. \tag{21}$$

The second term is the curvature term, whereas the third term is the Λ-term. It is easy to see that at large a the first term falls off faster than the curvature term which in turn falls off faster than the Λ-term. At the beginning, when $t = t_1$ and $a(t_1) = a_1$ we have

$$\rho(t_1) = -\rho_\Lambda(t_1) = -\frac{s^2(1-m)\Lambda^{2-m}m^{1-m}}{2^{2+m}3^{1-m}\pi G(1+m)^{2-m}},$$

therefore

$$\frac{\ddot{a}(t_1)}{a(t_1)} = -\frac{4\pi G}{3}\left(\rho(t_1) + 3\frac{p(t_1)}{c^2(t_1)}\right) + \frac{\Lambda c^2(t_1)}{3} = \frac{1}{3}\left(8\pi G\rho_\Lambda(t_1) + \frac{\Lambda s^2}{a_1^{2(1-m)}}\right) > 0.$$

[2] The starting value $z = 0$ means that the Universe tunneled from "nothing" to a closed universe of a finite radius $a_1 = z_1/\sqrt{\Lambda}$.

In other words, we have accelerated expansion at the beginning. As times goes by, the Λ-term will be dominating term so

$$a(t) \sim \left(s\sqrt{\frac{\Lambda(1-m)^3}{1+m}}t \right)^{1/(1-m)} \sim t^{\kappa},$$

with $\kappa = 1/(1-m) \in (1, 1.5)$, at $t \to \infty$. Therefore for $t \to \infty$ we have $\ddot{a} < 0$. It means that accelerated expansion continue during a finite time.

5 Peculiar cases with $n = -1$ and $n = -2$

Now, lets consider the cases of $n = -1$ and $n = -2$. The formula (14) is not valid in these cases ($m = -1$ and $m = 0$) so we shall consider these models separately.

If $n = -1$ ($m = -1$) then

$$\rho = \frac{M}{a^4} + \frac{\Lambda s^2}{8\pi G a^2} - \frac{3s^2}{4\pi G a^4} \log \frac{a}{a_*},$$

therefore

$$U(a) = s^2 \left(2\log\left(\frac{a}{a_*}\right) - \frac{2a^2\Lambda}{3} + 1 \right), \tag{22}$$

where a_* are constant and $[a_*]$=sm. The potential (22) has one maximum at $a = a_e = \sqrt{3/(2\Lambda)}$ such that $U_e = U(a_e) = 2s^2 \log(a_e/a_*)$, so if $a_e > a_*$ then $U_e > 0$ (see Fig. 4) . We choose $a_* = \Lambda^{-1/2}$. This gives us $U_e = 0.41s^2 > 0$. For the case $E \ll U_e$ the semiclassical tunneling nucleation probability is

$$P_{_{-1}} \sim \exp\left(-\frac{s^3\sqrt{\Lambda}}{4\pi G\hbar} \int_{z_i'}^{z_i} \frac{dz}{z^2} \sqrt{\log z^2 - \frac{2z^2}{3} + 1} \right) \sim \exp\left(-\frac{s^3\sqrt{\Lambda}}{10\pi G\hbar} \right), \tag{23}$$

where the turning points are $z_i' = 0.721$, $z_i = 1.812$. As we can see from the (23), when $n = -1$ we receive the aforementioned effect again.

If $n = -2$ ($m = 0$) then

$$\rho = \frac{M}{a^4} + \frac{s^2\Lambda}{2\pi G a^4} \log\left(\frac{a}{a_*}\right) + \frac{3s^2}{4\pi G a^6}.$$

We choose $a_* = 1/(\alpha\sqrt{\Lambda})$, where α is a dimensionless quantity. Thus

$$U(a) = -s^2 \left(\frac{1}{a^2} + \frac{4\Lambda}{3} \log\left(\alpha a\sqrt{\Lambda}\right) + \frac{\Lambda}{3} \right). \tag{24}$$

The maximum of potential (24) is located at the same point a_e and

$$U_e = -\frac{s^2\Lambda}{3} \left(3 + \log\left(\frac{9\alpha^4}{4}\right) \right).$$

Therefore, $U_e > 0$ if $\alpha < 2\mathrm{e}^{-3/4}/\sqrt{6} \sim 0.386$. Choosing $\alpha = 0.286$ and $E \ll U_e$ gets us the turning points $z_i' \sim 0.77$ and $z_i \sim 2.391$. The potential is pictured on the Fig.5.

At last, the semiclassical tunneling nucleation probability is

$$P_0 \sim \exp\left(-\frac{s^3\Lambda^2}{4\pi G\hbar} \int_{z_i'}^{z_i} \frac{dz}{z^4} \sqrt{-\frac{1}{z^2} - \frac{4}{3}\log(\alpha z) - \frac{1}{3}} \right) \sim \exp\left(-\frac{0.084s^3\Lambda^2}{\pi G\hbar} \right).$$

6 Conclusion

As we have shown, the semiclassical tunneling nucleation probability in VSL quantum cosmology is quit different from the one in quantum cosmology with c=const. In the first case this probability is strongly suppressed for large values of Λ whereas in the second case it is strongly suppressed for small values of Λ. Nevertheless, we can't really say that VSL quantum cosmology results in solution of the Λ-mystery. The problem is the validity the WKB wave function. And what is more, we have omitted all preexponential factors which can be essential ones near the turning points.

But, all in all, the difference between P in VSL and usual quantum cosmology seems very interesting and very significant.

ACKNOWLEDGMENTS

After finishing this work, we learned that T.Harko, H.Q.Lu, M.K.Mak and K.S.Cheng [10], have independently considered the VSL tunneling probability in both Vilenkin and Hartle-Hawking approaches. The interesting conclusion of their work is that at zero scale factor the classical singularity is no longer isolated from the Universe by the quantum potential but instead classical evolution can start from arbitrarily small size. In contrast to [10], we attract attention to the fact that the semiclassical tunneling nucleation probability in VSL quantum cosmology is strongly suppressed for large values of Λ and small values of the initial scale factor $a_i \sim 1\sqrt{\Lambda}$.

We'd like to thank Professor Harko for useful information about the article [10].

Appendix

Lets take $n = -2 - m$, for the $m \geq 0$. Substituting (21) into the first equation of system (1) yields

$$\ddot{a} = \frac{1}{a^3}\left[-E + \frac{s^2}{a^{2m}}\left(-\frac{m+2}{(m+1)a^2} + \frac{2(m+1)\Lambda}{3m}\right)\right]. \qquad (A1)$$

Thus we have the following situation:

1. If

$$0 < a^2 \ll \frac{3m(m+2)}{2\Lambda(m+1)^2},$$

then the curvature term is the dominating one and $\ddot{a} < 0$.

2. If

$$\frac{3m(m+2)}{2\Lambda(m+1)^2} \ll a^2 \ll \tilde{a}^2 \equiv \left(\frac{2s^2(m+1)\Lambda}{3mE}\right)^{1/m}, \qquad (A2)$$

then the dominating term is Λ-term and $\ddot{a} > 0$ during this time.

3. If

$$a^2 \gg \left(\frac{2s^2(m+1)\Lambda}{3mE}\right)^{1/m},$$

then the radiation term is the dominating one and $\ddot{a} < 0$.

There are two way to interpret the region (A2). The first way is to conclude that we have cosmological inflation in early universe. This is possible when $0 < m \ll 1$. In this

case we can evaluate the number of e-foldings ΔN during the region (A2) as

$$\log m \sim -2m\Delta N, \qquad \Lambda \gg \frac{3Em}{2s^2} \sim \frac{3Em}{2c_0^2 a_0^4}. \tag{A3}$$

If $\Delta N \sim 60$ then $m \sim 0.029$; if $\Delta N \sim 100$ then $m \sim 0.0197$. To evaluate E one can use the well-known expression for the Friedmann integrals [9],

$$A(w) = \left[\left(\frac{1+3w}{2}\right)^2 E\right]^{1/(1+3w)}.$$

Since $A(1/3) = 3 \times 10^{36}$ sm^2/sec, we get $E = 0.9 \times 10^{73}$ sm^4/sec^2. The substitution of $A(1/3)$ into the (A3) results in

$$\Lambda \gg 0.435 \times 10^{-61}\ \text{sm}^{-2}, \qquad \Lambda \gg 0.296 \times 10^{-61}\ \text{sm}^{-2},$$

for the $\Delta N = 60$ and $\Delta N = 100$.

But do we really need inflation in the VSL models? The question is not quite clear. On the one hand, VSL models can solve fundamental cosmological problems (horizon and flatness problems) without inflation - and what is more, these models can solve Λ-problem whereas inflations can't do it without the anthropic principle. On the other hand, the simplest case of VSL cosmological models, which is the subject of this article, is facing with the isotropy problem [3]. But, as we have seen, VSL model results in inflation with exit naturally so it will be incorrectly to oppose VSL models and the inflation.

Another way to interpret the region (A2) is to identify this region with modern acceleration of universe. This is possible if m is sufficiently large. Let us make a crude guess. According to modern observations we can write $\ddot{a}_0/a_0 = 5.6\pi G\rho_c/3$ where $\rho_c = 10^{-29}$ gramme/sm^3. If the modern value of $a_0 \sim \tilde{a}$ (see the inequality (A2)) then

$$\Lambda = \frac{3mE}{3c_0^2(m+1)a_0^4}. \tag{A4}$$

From the (A1) we have

$$\ddot{a}_0 \sim \frac{2(m+1)\Lambda s^2}{3m a_0^{2m+3}}$$

if the Λ-term is dominating one. Substituting (A4) gets us

$$\Omega_{\Lambda} = \frac{\Lambda c_0^2}{8\pi G\rho_c} \sim \frac{0.35m}{m+1}, \qquad a_0 \sim \sqrt[4]{\frac{3E}{5.6\pi G\rho_c}}.$$

Thus, if $m \gg 1$ then $\Omega_{\Lambda} \sim 0.35$, $\Lambda \sim 0.2 \times 10^{-56}$ sm^{-2}, $a_0 \sim 10^{27}$ sm. And these values are seems to be quite reasonable.

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